

**INTRO TO NUMBER THEORY: HW1**  
**DUE THURSDAY 9/20/2018 (IN CLASS)**

**Question 1.** *Prove that if  $n$  is an odd integer, then  $n^2 - 1$  is divisible by 8.*

**Question 2.** *Prove that  $(a, a + 2) = 1$  or  $2$  for every integer  $a$ .*

**Question 3.** *Suppose  $p > 0$  is a prime number such that  $p = 3k + 1$  for some integer  $k$ . Show that  $p = 6k' + 1$  for some integer  $k'$ .*

**Question 4.** *Evaluate  $(ab, p^4)$  and  $(a + b, p^4)$  given that  $(a, p^2) = p$  and  $(b, p^3) = p^2$ . Provide a short justification for your answer.*

**Question 5.** *Suppose  $a$ ,  $b$  and  $c$  are integers with  $(a, b) = c$ . Show that  $(a^2, b^2) = c^2$ .*

**Question 6.** *A pair of twin primes is a tuple  $(p, p + 2)$  with both  $p$  and  $p + 2$  prime. Show that there is a bijective correspondence between pairs of twin primes and integers  $n > 0$  such that  $n^2 - 1$  has exactly 4 positive divisors (including 1 and  $n^2 - 1$ ).*