

GRAPH THEORY HW 3: DUE (IN CLASS) WEDNESDAY 4/18/2018

DAVID J. GRYNKIEWICZ

Question 1. Let $G = (V, E)$ be a loopless, connected graph on $n \geq 2$ vertices. Recall that an edge cut is a subset $E' \subseteq E$ such that $G \setminus E'$ (the graph with the edges from E' removed) is disconnected, and a vertex cut is a subset $V' \subseteq V$ such that the graph $G \setminus V'$ (the graph induced by $V \setminus V'$) is disconnected.

1. If $E' \subseteq E$ is a minimal edge cut, meaning $|E'|$ is minimal over all possible edge cuts, show that $E' = \delta_G(X)$ for some $X \subseteq V$, where $\delta_G(X) \subseteq E$ consists of all edges in G having exactly one of their two endpoints in X .
2. If $Z \subseteq V$ is a minimal vertex cut, meaning $|Z|$ is minimal over all possible vertex cuts, and $X \subseteq G \setminus Z$ is the vertex set of a connected component of $G \setminus Z$, show that $Z = N_G(X) \setminus X$, where $N_G(X) \subseteq V$ denotes all vertices adjacent to some vertex from X .

Question 2. For each of the networks $N(x, y)$ labelled A, B and C in the addendum, mark the edges in a minimal capacity edge cut and label the edges so as to give a maximal value flow.

Question 3. For each of the networks $N(x, y)$ labelled D, E and F in the addendum, which have vertex capacity restrictions instead of edge capacity restrictions, mark the vertices in a minimal cost vertex cut and label the edges so as to give a maximal value flow subject to the vertex capacity constraints.

Question 4. An international company has employees from three countries, France, Germany and England. At the main head quarters, there are 33 task forces devoted to special projects. Each employee serves on one or more of the task forces. It is desired to form a council consisting of one selected member from each task force. However, no one employee should represent more than one task force and there should be the same number of people from each of the three countries serving on the council. Explain how to construct a network which can be used to determine if it is possible to select members for the council according to the above restrictions and, if it is, find one possible choice of representatives meeting the requirements. Hint: flow should correspond to employees selected