GRAPH THEORY HW 1: DUE (IN CLASS) WEDNESDAY 3/21/2018

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Question 1. *Show that* $\chi(C_n) = 3$ *for* $n \geq 3$ *odd.*

Question 2. *Determine the Chromatic number of a tree.*

Question 3. *Let* $G = (V, E)$ *be a simple graph with* $k = \chi(G)$*. For any proper* k*-coloring* $c : V \to \{1, 2, \ldots, k\}$*, and for any* $r \in \{1, 2, \ldots, k\}$*, there exists a vertex colored by* r *which is adjacent to vertices of every other color.*

Question 4. *Let* $G = (V, E)$ *be a simple graph. Show that* G *has at least* $\chi(G)$ *vertices whose degrees are each at least* $\chi(G) - 1$*.*

Question 5. *Let* $G = (V, E)$ *be a simple graph with* $x_1, \ldots, x_n \in V$ *its distinct* n *vertices and* $d_1 = d(x_1), d_2 = d(x_2), \ldots d_n = d(x_n)$ *the sequence of degrees of vertices in* G *ordered so that* $d_1 \geq d_2 \geq \ldots \geq d_n$*. Show that*

$$\chi(G) \leq \max\{\min\{d_i + 1, i\} : 1 \leq i \leq n\}.$$

Question 6. *Let* G *be a simple graph with* m *edges. Show that* $\chi(G) \leq \lceil \sqrt{2m} \rceil$